Canonical analysis of space-time noncommutative theories and gauge symmetries

Dmitri V. Vassilevich[*]

Institut für Theoretische Physik, Universität Leipzig,
Augustusplatz 10, D-04109 Leipzig, Germany
email: `Dmitri.Vassilevich@itp.uni-leipzig.de`

Abstract

We construct a modification of the Poisson bracket which is suitable for a canonical analysis of space-time noncommutative field theories. We show that this bracket satisfies the Jacobi identities and generates equations of motion. In this modified canonical formalism one can define the notion of the first-class constraints, demonstrate that they generate gauge symmetries, and derive an explicit form of these symmetry transformations.

1 Introduction

Among all noncommutative field theories (cf. reviews [1]) the theories with space-time noncommutativity have a somewhat lower standing since it is believed that they cannot be properly quantised because of the problems with causality and unitarity (see, e.g., [2]). Such problems occur due to the time-nonlocality of these theories caused by the presence of an infinite number of temporal derivatives in the Moyal star product. However, it has been shown later, that unitarity can be restored [3] (see also [4]) in space-time noncommutative theories and that the path integral quantization can be performed [5]. This progress suggests that space-time noncommutative theories may be incorporated in general formalism of canonical quantization [6]. Indeed, a canonical approach has been suggested in [7].

Apart from quantization, there is another context in which canonical approach is very useful. This is the canonical analysis of constraints and corresponding gauge

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[*]Also at Department of Physics, St. Petersburg State University, Russia.

symmetries [6]. The problem of symmetries becomes extremely complicated in noncommutative theories. Already at the level of global symmetries one see phenomena which never appear in the commutative theories. For example, the energy-momentum tensor in translation-invariant noncommutative theories is not locally conserved (cf. pedagogical comments in [8]). At the same time all-order renormalizable noncommutative ϕ^4 theory is *not* translation-invariant [9]. A Lorentz-invariant interpretation of noncommutative space-time leads to a twisted Poincare symmetry [10]. It is unclear how (and if) this global symmetry can be related to local diffeomorphism transformations analysed, e.g., in [11]. Proper deformation of gauge symmetries of generic two-dimensional dilaton gravities remains on open problem (cf. footnote 2). Solving (some of) the problems related to gauge symmetries in noncommutative field theories by the canonical methods is the main motivation for this work.

We start our analysis from the very beginning, i.e. with a definition of the canonical bracket. Our approach is based on two main ideas. First of all, we separate implicit time derivatives (which are contained in the Moyal star), and explicit ones (which survive in the commutative limit). Only explicit derivatives define the canonical structure. As a consequence, the constraints and the hamiltonian become non-local in time. Therefore, the notion of same-time canonical brackets becomes meaningless. We simply postulate a bracket between canonical variables taken at different points of space ($\mathbf{x}$ and $\mathbf{x}'$) and of time (t and t'):

$$\{q_a(\mathbf{x}, t), p^b(\mathbf{x}', t')\} = \delta_a^b \delta(\mathbf{x} - \mathbf{x}')\delta(t - t') \tag{1}$$

This bracket is somewhat similar to the one appearing in the Ostrogradski formalism for theories with higher order time derivatives (see, e.g., [12] for applications to field theories and [7] for the use in space-time noncommutative theories), but there are important differences (a more detailed comparison is postponed until sec. 4).

Of course, the proposed formalism means a departure from the standard canonical procedure. Nevertheless, we are able to demonstrate that the new bracket satisfies such fundamental requirements as antisymmetry and the Jacobi identities. These brackets generate equations of motion. Moreover, one can define the notion of first-class constraints with respect to the new bracket and show that these constraints generate gauge symmetries of the action. We shall derive an explicit form of the symmetry transformation and see that they look very similar to the commutative case (the only difference, in fact, is the modified bracket and the star product everywhere).

2 Canonical bracket

Consider a space-time manifold $\mathcal{M}$ of dimension D. Consider the Moyal product of functions on $\mathcal{M}$

$$f \star g = f(x) \exp\left(\frac{i}{2} \theta^{\mu\nu} \overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu\right) g(x). \tag{2}$$

In this form the star product has to be applied to plane waves and then extended to all (square integrable) functions by means of the Fourier series. θ is a constant antisymmetric matrix. We impose no restrictions on θ, i.e. we allow for the space-time noncommutativity.

The Moyal product is closed,

$$\int_{\mathcal{M}} d^D x f \star g = \int_{\mathcal{M}} d^D x f \times g \tag{3}$$

(where $\times$ denotes usual commutative product), it respects the Leibniz rule

$$\partial_\mu (f \star g) = (\partial_\mu f) \star g + f \star (\partial_\mu g), \tag{4}$$

and allows to make cyclic permutations under the integral

$$\int_{\mathcal{M}} d^D x f \star g \star h = \int_{\mathcal{M}} d^D x h \star f \star g \tag{5}$$

The phase space on $\mathcal{M}$ consists of the variables r_j which can be subdivided into canonical pairs q, p and other variables α which do not have canonical partners (these will play the role of Lagrange multipliers or of gauge parameters). We define a bracket (r_j, r_k) to be ± 1 on the canonical pairs,

$$(q_a, p^b) = -(p^b, q_a) = \delta_a^b \tag{6}$$

and zero otherwise (e.g., $(\alpha, p) = (p^a, p^b) = 0$). With this definition the bracket (1) reads: $\{r_i(x), r_j(x')\} = (r_i, r_j)\delta(x - x')$. Note, that we are not going to use brackets between two local expressions (see discussion below).

Now we can define canonical brackets between star-local functionals on the phase space. We define the space of star-local *expressions* as a suitable closure of the space of free polynomials of the phase space variables r_j and their derivatives evaluated with the Moyal star. Such expressions integrated over $\mathcal{M}$ we call star-local *functionals*.

Locality plays no important role here, since after the closure one can arrive at expressions with arbitrary number of explicit derivatives (besides the ones present implicitly through the Moyal star). It is important, that all expressions can be approximated with only one type of the product (namely, the Moyal one), and no

mixed expressions with star and ordinary products appear. One also has to define what does "suitable closure" actually mean, i.e. to fix a topology on the space of the functionals. This question is related to the restrictions which one imposes on the phase space variables. For example, the bracket of two admissible functionals (see (8) below) should be again an admissible functional. This implies that all integrands are well-defined and all integrals are convergent. Stronger restrictions on the phase space variables mean weaker restrictions on the functionals, and vice versa. Such an analysis cannot be done without saying some words about $\mathcal{M}$ (or about its' compactness, at least)[1]. We shall not attempt to do this analysis here (postponing it to a future work). All statements made below are true at least for $r \in C^\infty$ and for polynomial functionals (no closure at all).

Obviously, it is enough to define the bracket on monomial functionals and extend it to the whole space by the linearity. Generically, two such monomial functionals read:

$$R = \int d^D x \, \partial_{\kappa_1} r_1 \star \partial_{\kappa_2} r_2 \star \ldots \partial_{\kappa_n} r_n \,, \qquad \tilde{R} = \int d^D x \, \partial_{\tilde{\kappa}_1} \tilde{r}_1 \star \partial_{\tilde{\kappa}_2} \tilde{r}_2 \star \ldots \partial_{\tilde{\kappa}_m} \tilde{r}_m \quad (7)$$

κ_j is a multi-index, ∂_{κ_j} is a differential operator of order $|\kappa_j|$. The (modified) canonical bracket of two monomials is defined by the equation

$$\{R, \tilde{R}\} = \sum_{i,j} \int d^D x \, \partial_{\kappa_j} \left(\partial_{\kappa_{j+1}} r_{j+1} \star \ldots \partial_{\kappa_{j-1}} r_{j-1} \right) (r_j, \tilde{r}_i)$$

$$\star \partial_{\tilde{\kappa}_i} \left(\partial_{\tilde{\kappa}_{i+1}} \tilde{r}_{i+1} \star \ldots \partial_{\tilde{\kappa}_{i-1}} \tilde{r}_{i-1} \right) (-1)^{|\kappa_j| + |\tilde{\kappa}_i|} \,. \qquad (8)$$

In other words, to calculate the bracket between two monomials one has to (i) take all pairs r_j, $\tilde{r}_i$; (ii) use cyclic permutations under the integrals to move r_j to the last place, and $\tilde{r}_i$ – to the first; (iii) integrate by parts to remove derivatives from r_j and $\tilde{r}_i$; (iv) delete r_j and $\tilde{r}_i$, put the integrands one after the other connected by $\star$ and multiplied by $(r_j, \tilde{r}_i)$; (v) integrate over $\mathcal{M}$. Actually, this is exactly the procedure one uses in usual commutative theories modulo ordering ambiguities following from the noncommutativity.

The following Theorem demonstrates that the operation we have just defined gives indeed a Poisson structure on the space of star-local functionals.

Theorem 2.1 *Let R, $\tilde{R}$ and $\hat{R}$ be star-local functionals on the phase space. Then*
(1) $\{R, \tilde{R}\} = -\{\tilde{R}, R\}$ (antisymmetry),
(2) $\{\{R, \tilde{R}\}, \hat{R}\} + \{\{\hat{R}, R\}, \tilde{R}\} + \{\{\tilde{R}, \hat{R}\}, R\} = 0$ (Jacobi identity).

Proof. We start with noting that since we do not specify the origin of the canonical variables, the time coordinate does not play any significant role, and the statements

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[1]Some restrictions on $\mathcal{M}$ follow already from the existence of the Moyal product, which requires existence of a global coordinate system at least in the noncommutative directions.

above (almost) follow from the standard analysis [6]. However, it is instructive to present here a complete proof as it shows that one do not need to rewrite the star product through infinite series of derivatives (so that the $\star$ product indeed plays a role of multiplication). Again, it is enough to study the case when all functionals are monomial ones. Then the first assertion follows from (8) and $(r_j, r_k) = -(r_k, r_j)$. Let

$$\hat{R} = \int d^D x \, \partial_{\hat{\kappa}_1} \hat{r}_1 \star \partial_{\hat{\kappa}_2} \hat{r}_2 \star \dots \partial_{\hat{\kappa}_p} \hat{r}_p \,. \tag{9}$$

Consider $\{\{R, \tilde{R}\}, \hat{R}\}$. The first of the brackets "uses up" an r_j and an $\tilde{r}_i$. The second bracket uses a variable with hat and another variable either from R or from $\tilde{R}$. Consider first the terms in the repeated bracket which use twice some variables from R. All such terms combine into the sum

$$\sum_{i,k,j \neq l} (-1)^{|\tilde{\kappa}_i| + |\hat{\kappa}_k|} (r_j, \tilde{r}_i)(r_l, \hat{r}_k) \int d^D x \, \partial_{\kappa_{l+1}} r_{l+1} \star \dots \partial_{\kappa_{j-1}} r_{j-1}$$
$$\star \partial_{\kappa_j + \tilde{\kappa}_i} \left(\partial_{\tilde{\kappa}_{i+1}} \tilde{r}_{i+1} \star \dots \partial_{\tilde{\kappa}_{i-1}} \tilde{r}_{i-1} \right) \star \partial_{\kappa_{j+1}} r_{j+1} \star \dots \partial_{\kappa_{l-1}} r_{l-1}$$
$$\star \partial_{\kappa_l + \hat{\kappa}_k} \left(\partial_{\hat{\kappa}_{k+1}} \hat{r}_{k+1} \star \dots \partial_{\hat{\kappa}_{k-1}} \hat{r}_{k-1} \right)$$

This complicated expression is symmetric with respect to interchanging the roles of the variables with hats and the variables with tilde. Therefore, it is clear that the terms having two brackets with r in $\{\{\hat{R}, R\}, \tilde{R}\}$ have exactly the same form as above but with a minus sign. No such terms (with two brackets with r) may appear in $\{\{\tilde{R}, \hat{R}\}, R\}$. Therefore, this kind of terms are cancelled in $\{\{R, \tilde{R}\}, \hat{R}\} + \{\{\hat{R}, R\}, \tilde{R}\} + \{\{\tilde{R}, \hat{R}\}, R\}$. By repeating the same arguments for $\hat{r}$ and $\tilde{r}$ one proves our second assertion. $\square$

One can define a canonical bracket between functionals and densities (star-local expressions) by the equation:

$$\{R, h(r)(x)\} := \frac{\delta}{\delta \beta(x)} \left\{ R, \int d^D y \, \beta(y) \star h(r)(y) \right\} \,. \tag{10}$$

To construct brackets between two densities (i.e., to give a proper extension of (1) to nonlinear functions) one has to define star-products with delta-functions which may be a very non-trivial task. We shall never use brackets between densities.

To use the canonical bracket in computations of variation we need the following technical Lemma.

Lemma 2.2 *Let p^a and q_b depend smoothly on a parameter τ. We assume that the variables $\alpha(x)$ (these are the ones which do not have canonical conjugates) do not*

depend on τ. Let $h(r(\tau))$ be a star-local expression on the phase space. Then

$$\partial_\tau \int d^D x\, \beta \star h(r(\tau)) = \int d^D x\, \left((\partial_\tau q_a) \star \left\{ \int d^D y\, \beta \star h(r), p^a(x) \right\} \right.$$
$$\left. -(\partial_\tau p^a) \star \left\{ \int d^D y\, \beta \star h(r), q_a(x) \right\} \right) \qquad (11)$$

Proof. Obviously, it is enough to prove this Lemma for $\beta = 1$. Let us consider first the case when just one of the canonical variables (say, p^b for a just single value of b) depends on τ, and when $h(r) = h_1(r) \star \partial_\kappa p^b \star h_2(r)$ where neither h_1 nor h_2 depend on p^b. Then

$$\partial_\tau \int d^D x\, h(r) = \int d^D x\, h_1 \star \partial_\kappa (\partial_\tau p^b) \star h_2 = (-1)^{|\kappa|} \int d^D x\, \partial_\kappa (h_2 \star h_1) \star \partial_\tau p^b \,. \qquad (12)$$

On the other hand, by using (8), one obtains

$$\left\{ \int d^D x\, h(r), \int d^D y\, \beta(y) \star q_b(y) \right\} = -(-1)^{|\kappa|} \int d^D x\, \partial_\kappa (h_2 \star h_1) \star \beta \,. \qquad (13)$$

Next we use (10) to see that the statement of this Lemma is indeed true for the simplified case considered. In general case one has to sum up many individual contributions to both sides of (11) from different canonical variables occupying various places in h. Each of this contributions can be treated in the same way as above. $\square$

As an application, consider a noncommutative field theory described by the action

$$S = \int (p^a \partial_t q_a - h(p, q, \lambda))\, d^D x = \int p^a \partial_t q_a d^D x - H \,, \qquad (14)$$

where h is a star-local expression, it contains temporal derivatives only implicitly, i.e. only though the Moyal star. Note, that due to (3) the star between p^a and $\partial_t q_a$ can be omitted. If one takes into account explicit time derivatives only, one can write $p^a = \delta S/(\delta \partial_t q_a)$. Then, $H = S - \int p \partial_t q d^D x$.

The equations of motion generated from the action (14) by taking variations with respect to q and p can be written in the "canonical" form:

$$\partial_t p^a + \{H, p^a\} = 0 \,, \qquad \partial_t q_a + \{H, q_a\} = 0 \qquad (15)$$

This can be easily shown by taking $q(\tau) = q + \tau \delta q$ and $p(\tau) = p + \tau \delta p$ and using Lemma 2.2. No explicit time derivative acts on λ. In a commutative theory λ generates constraints.

3 Constraints and gauge symmetries

Let us specify the form of (14):

$$S = \int \left(p^a \partial_t q_a - \lambda^j \star G_j(p,q) - h(p,q) \right) d^D x \tag{16}$$

We shall call $G_j(p,q)$ a constraint, although due to the presence of the Moyal star it cannot be interpreted as a condition on a space-like surface. Dirac classification of the constraints can be also performed with the modified canonical bracket. We say that the constraints $G_j(p,q)$ are first-class if their brackets with $h(p,q)$ and between each other are again constraints, i.e.,

$$\left\{ \int d^D x \alpha^i \star G_i, \int d^D x \beta^j \star G_j \right\} = \int d^D x C(p,q;\alpha,\beta)^k \star G_k, \tag{17}$$

$$\left\{ \int d^D x \alpha^i \star G_i, \int d^D x h(p,q) \right\} = \int d^D x B(p,q;\alpha)^k \star G_k. \tag{18}$$

By Theorem 2.1(1) the structure functions are antisymmetric, $C(p,q;\alpha,\beta)^j = -C(p,q;\beta,\alpha)^j$. Further restrictions on C and B follow from the Jacobi identities (cf. Theorem 2.1(2)).

Theorem 3.1 *Let $G_i(p,q)$ be fist-class constraints (so that (17) and (18) are satisfied). Then the transformations*

$$\delta p^a = \left\{ \int d^D x \alpha^j \star G_j, \; p^a \right\} \tag{19}$$

$$\delta q_b = \left\{ \int d^D x \alpha^j \star G_j, \; q_b \right\} \tag{20}$$

$$\delta \lambda^j = -\partial_t \alpha^j - C(p,q;\alpha,\lambda)^j - B(p,q;\alpha)^j \tag{21}$$

with arbitrary α^j are gauge symmetries of the action (16).

Proof. To prove this Theorem we simply check invariance of (16) under (19) - (21). Let $f(p,q)$ be an arbitrary star-local expression depending on the canonical variables p and q only. Then, by (19) and (20),

$$\delta f(p,q) = \left\{ \int d^D x \alpha^j \star G_j, \; f(p,q) \right\} \tag{22}$$

It is now obvious that that the transformations of G and h in the action (16) are compensated by second and third terms in $\delta\lambda$ respectively. The remaining term in

the action transforms as

$$
\delta \int d^D x \, p^a \partial_t q_a =
$$

$$
= \int d^D x \left(\left\{ \int d^D y \alpha^j \star G_j, p^a(x) \right\} \star \partial_t q_a + p^a \star \partial_t \left\{ \int d^D y \alpha^j \star G_j, q_a(x) \right\} \right)
$$

$$
= \int d^D x \left(\left\{ \int d^D y \alpha^j \star G_j, p^a(x) \right\} \star \partial_t q_a - (\partial_t p^a) \star \left\{ \int d^D y \alpha^j \star G_j, q_a(x) \right\} \right)
$$

$$
= \int d^D x \alpha^j \star \partial_t G_j = - \int d^D x \partial_t(\alpha^j) \star G_j \tag{23}
$$

Here we used integration by parts and Lemma 2.2. The last term in (23) is compensated by the first (gradient) term in the variation (21). Therefore, the action (16) is indeed invariant under (19) - (21). $\square$

Example. Consider a two-dimensional topological noncommutative theory described by the action

$$
S = \int \mathrm{tr} \left(\Phi \star F \right) = \int d^2 x \, \mathrm{tr} \left(\Phi \star F_{01} \right) \tag{24}
$$

F is a field-strength two-form with the components: $F_{01} = \partial_0 A_1 - \partial_1 A_0 + [A_0, A_1]$. All commutators are taken with the Moyal star. The scalar field Φ and the vector potential A_μ are matrix-valued. They can be expanded over generators the T_J of a Lie group taken in an appropriate representation, $\Phi = \Phi^J T_J$, $A_\mu = A_\mu^J T_J$. If the gauge group is $U(1,1)$ this model is equivalent to a noncommutative version [13] of the Jackiw-Teitelboim gravity [14][2]. We suppose that the quadratic form $\eta_{IJ} = -\mathrm{tr}(T_I T_J)$ is non-degenerate, and that the algebra is closed with respect to commutators and anti-commutators:

$$
[T_I, T_J] = f_{IJ}{}^K T_K, \qquad [T_I, T_J]_+ = T_I T_J + T_I T_J = d_{IJ}{}^K T_K . \tag{25}
$$

The action (24) can be rewritten as

$$
S = \int d^2 x \left(-\eta_{IJ} \Phi^I \star \partial_0 A_1^J - A_0^I \eta_{IJ} \star \left(\partial_1 \Phi^J + \frac{1}{2} d_{KL}{}^J [A_1^K, \Phi^L] \right. \right.
$$

$$
\left. \left. + \frac{1}{2} f_{KL}{}^J [A_1^K, \Phi^L]_+ \right) \right) \tag{26}
$$

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[2] This model is the only two-dimensional dilaton gravity so far which has a noncommutative counterpart. It is an interesting and important task to deform other 2D gravities [15] as well. Constructing and appropriate classical action with right number of gauge symmetries seems to be the most hard part of the problem. Quantization goes then [16] as in the commutative case [17], at least for the model [13].

According to this action, the canonical variables are $q^I = A_i^I$ and $p_I = -\eta_{IJ}\Phi^J$. Lagrange multipliers A_0^I generate the constraints:

$$G_I = \eta_{IJ}\left(\partial_1\Phi^J + \frac{1}{2}d_{KL}{}^J[A_1^K, \Phi^L] + \frac{1}{2}f_{KL}{}^J[A_1^K, \Phi^L]_+\right).\tag{27}$$

The structure functions

$$C(\alpha, \beta)^L = -\frac{1}{2}\left(d_{IJ}{}^L[\alpha^I, \beta^J] + f_{IJ}{}^L[\alpha^I, \beta^J]_+\right)\tag{28}$$

do not depend on the canonical variables. The gauge transformations (cf Theorem 3.1)

$$\delta\Phi^J = -\frac{1}{2}d_{LI}{}^J[\Phi^L, \alpha^I] - \frac{1}{2}f_{LI}{}^J[\Phi^L, \alpha^I]_+\tag{29}$$

$$\delta A_\mu^J = -\partial_\mu\alpha^J - \frac{1}{2}d_{LI}{}^J[A_\mu^L, \alpha^I] - \frac{1}{2}f_{LI}{}^J[A_\mu^L, \alpha^I]_+\tag{30}$$

are just infinitesimal versions of usual noncommutative gauge transformations $\Phi \to e^{\alpha T}\star\Phi\star e^{-\alpha T}$, $A_\mu \to e^{\alpha T}\star\partial_\mu e^{-\alpha T} + e^{\alpha T}\star A_\mu\star e^{-\alpha T}$. The exponentials are calculated with the star-product.

In a space-space noncommutative theory similar calculations were done in [18].

4   Conclusions

In this paper we have suggested a modification of the Poisson bracket which is defined on fields at different values of the time coordinate. In this modified canonical formalism, only explicit time derivatives (i.e., the ones which are not hidden in the Moyal multiplication) define the canonical structure. Although this means serious deviations from standard canonical methods, the resulting brackets still satisfy the Jacobi identities (Theorem 2.1) and generate classical equations of motion. Our main result (Theorem 3.1) is that we can still define the notion of fist-class constraints, which generate gauge symmetries, and these symmetries are written down explicitly[3].

Let us compare the technique developed here to the Ostrogradski formalism for theories with higher time derivatives. In this formalism [12, 7] new phase space variables $P(t, T) = p(t + T)$ and $Q(t, T) = q(t + T)$ are introduced. Then t is interpreted as an evolution parameter, while T labels degrees of freedom (number

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[3]Just existence of the symmetries does not come as a great surprise in the view of the analysis of [19] which is valid for theories with arbitrary (but finite!) order of time derivatives. An important feature of the present approach is rather simple explicit formulae similar to that in the case of commutative theories with 1st order time derivatives.

of degrees of freedom is proportional to the order of temporal derivatives). Then a delta-function $\delta(T - T')$ appears naturally on the right hand side of the Poisson brackets between Q and P calculated at the same value of t. By returning (naively) to the original variables q and p one obtains (1). In the approach of [7] one proceeds in a different way. The resulting dynamical system is interpreted as a system with an infinite number of second-class constraints. Additional first-class constraints would lead to considerable complications in this procedure. It may happen that these two approaches are equivalent, but this requires further studies.

As for the prospects of the formalism developed in the present paper one should mention first of all applications to particular physical systems briefly outlined above. A more formal development would be to construct a classical BRST formalism starting with our brackets. Anyway, it is important to restore the reputation of space-time noncommutative theories. This is required by the principles of symmetry between space and time, but also by interesting physical phenomena which appear due to the space time noncommutativity (just as an example we may mention creation of bound states with hadron-like spectra [20]).

Acknowledgments

References

[1] Douglas M. R. and Nekrasov N. A., Noncommutative field theory, Rev. Mod. Phys. **73** (2001) 977-1029 [arXiv:hep-th/0106048];

Szabo R. J., Quantum field theory on noncommutative spaces, Phys. Rept. **378** (2003) 207-299 [arXiv:hep-th/0109162].

[2] N. Seiberg, L. Susskind and N. Toumbas, Space/time non-commutativity and causality, JHEP **0006** (2000) 044 [arXiv:hep-th/0005015];

J. Gomis and T. Mehen, Space-time noncommutative field theories and unitarity, Nucl. Phys. B **591** (2000) 265 [arXiv:hep-th/0005129].

L. Alvarez-Gaume and J. L. F. Barbon, Non-linear vacuum phenomena in non-commutative QED, Int. J. Mod. Phys. A **16** (2001) 1123 [arXiv:hep-th/0006209].

C. S. Chu, J. Lukierski and W. J. Zakrzewski, Hermitian analyticity, IR/UV mixing and unitarity of noncommutative field theories, Nucl. Phys. B **632** (2002) 219 [arXiv:hep-th/0201144].

[3] D. Bahns, S. Doplicher, K. Fredenhagen and G. Piacitelli, On the unitarity problem in space/time noncommutative theories, Phys. Lett. B **533** (2002) 178 [arXiv:hep-th/0201222].

C. h. Rim and J. H. Yee, Unitarity in space-time noncommutative field theories, Phys. Lett. B **574** (2003) 111 [arXiv:hep-th/0205193].

Y. Liao and K. Sibold, Time-ordered perturbation theory on noncommutative spacetime. II. Unitarity, Eur. Phys. J. C **25** (2002) 479 [arXiv:hep-th/0206011].

[4] A. P. Balachandran, T. R. Govindarajan, C. Molina and P. Teotonio-Sobrinho, Unitary quantum physics with time-space noncommutativity, arXiv:hep-th/0406125.

[5] K. Fujikawa, Path integral for space-time noncommutative field theory, arXiv:hep-th/0406128.

[6] D. M. Gitman and I. V. Tyutin, *Quantization of Fields with Constraints* (Springer-Verlag, Berlin 1990)

M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton 1992)

[7] J. Gomis, K. Kamimura and J. Llosa, Hamiltonian formalism for space-time non-commutative theories, Phys. Rev. D **63** (2001) 045003 [arXiv:hep-th/0006235].

[8] A. Gerhold, J. Grimstrup, H. Grosse, L. Popp, M. Schweda and R. Wulkenhaar, The energy-momentum tensor on noncommutative spaces: Some pedagogical comments, arXiv:hep-th/0012112.

[9] H. Grosse and R. Wulkenhaar, Renormalisation of ϕ^4 theory on noncommutative $\mathbb{R}^4$ to all orders, arXiv:hep-th/0403232.

[10] M. Chaichian, P. Kulish, K. Nishijima and A. Tureanu, On a Lorentz-invariant interpretation of noncommutative space-time and its implications on noncommutative QFT, arXiv:hep-th/0408069.

[11] R. Jackiw and S. Y. Pi, Covariant coordinate transformations on noncommutative space, Phys. Rev. Lett. **88** (2002) 111603 [arXiv:hep-th/0111122].

[12] R. Marnelius, The Lagrangian And Hamiltonian Formulation Of Relativistic Particle Mechanics, Phys. Rev. D **10** (1974) 2535

J. Llosa and J. Vives, Hamiltonian formalism for nonlocal Lagrangians, J. Math. Phys. **35** (1994) 2856

R. P. Woodard, A canonical formalism for Lagrangians with nonlocality of finite extent, Phys. Rev. A **62** (2000) 052105 [arXiv:hep-th/0006207].

[13] S. Cacciatori, A. H. Chamseddine, D. Klemm, L. Martucci, W. A. Sabra and D. Zanon, Noncommutative gravity in two dimensions, Class. Quant. Grav. **19** (2002) 4029 [arXiv:hep-th/0203038].

[14] C. Teitelboim, Gravitation And Hamiltonian Structure In Two Space-Time Dimensions, Phys. Lett. B **126** (1983) 41; The Hamiltonian Structure of Two-Dimensional Space-Time and its Relation with the Conformal Anomaly, in Quantum Theory Of Gravity, p. 327-344, S.Christensen (ed.), (Adam Hilgar, Bristol, 1983);

R. Jackiw, Liouville Field Theory: A Two-Dimensional Model For Gravity? in Quantum Theory Of Gravity, p. 403-420, S.Christensen (ed.) (Adam Hilgar, Bristol, 1983); Lower Dimensional Gravity, Nucl. Phys. B **252** (1985) 343.

[15] D. Grumiller, W. Kummer and D. V. Vassilevich, Dilaton gravity in two dimensions, Phys. Rept. **369** (2002) 327 [arXiv:hep-th/0204253].

[16] D. V. Vassilevich, Quantum noncommutative gravity in two dimensions, arXiv:hep-th/0406163.

[17] W. Kummer, H. Liebl and D. V. Vassilevich, Exact path integral quantization of generic 2-D dilaton gravity, Nucl. Phys. B **493** (1997) 491 [arXiv:gr-qc/9612012].

[18] R. Banerjee, Noncommuting electric fields and algebraic consistency in noncommutative gauge theories, Phys. Rev. D **67** (2003) 105002 [arXiv:hep-th/0210259].

[19] D. M. Gitman and I. V. Tyutin, Symmetries in Constrained Systems, arXiv: hep-th/0409087.

[20] D. V. Vassilevich and A. Yurov, Space-time non-commutativity tends to create bound states, Phys. Rev. D **69** (2004) 105006 [arXiv:hep-th/0311214].